Formal Description of Autopoiesis for Analytic Models of Life and Social Systems

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Abstract

Since the concept of autopoiesis was proposed as a model of minimal living systems by Maturana and Varela, and applied to social systems by Luhmann, there has been still few mathematically strict models to represent the characteristics of it because of its difficulty for interpretation. In order to verify the validity of this concept, this paper proposes a formal description of autopoiesis based on the theory of category and Rosen's perspective of "closure under efficient cause", provides analytic models of life and social systems, and discusses the effectiveness of autopoiesis in systems sciences through implication from the model.

Introduction

In order to consider possibility of realization of artificial life, we should discuss whether life systems themselves and societies as collective phenomena of life systems are able to be formalized within the conventional formal theories. Autopoiesis (Maturana & Varela 1980; Kneer & Nassehi 1993) is one of the most important meterials for this discussion.

Autopoiesis gives a framework in which a system exists as an organism through physical and chemical processes, based on the assumption that organisms are machinery (Maturana & Varela 1980). According to the original definition of it by Maturana and Varela, an autopoietic system is one that continuously produces the components that specify it, while at the same time realizing itself to be a concrete unity in space and time; this makes the network of production of components possible. An autopoietic system is organized as a network of processes of production of components, where these components (1) continuously regenerate and realize the network that produces them, and (2) constitute the system as a distinguishable unity in the domain in which they exist. However, there has been still few mathematically strict models that represent autopoiesis. In (Nomura 1997; 2000), we discussed the difficulty of interpreting autopoiesis within system theories using state spaces and problems of some models proposed for representing autopoiesis.

The aim in this paper is to discuss possibility of formal description of life and social systems by clarifying whether autopoiesis can really be represented within more abstract mathematical frameworks. For this aim, we use the theory of category (Takeuchi 1978), one of the most abstract algebraic structure representing relations between components. In this paper, we propose another closed systems by extending Rosen's (M,R) systems (Rosen 1972), and coupling of these closed systems for a possibility to formally represent Luhmann's social system (Kneer & Nassehi 1993).

Closure under Entailment

Rosen compared machine systems with living systems to clarify the difference between them, based on the relationship among components through entailment (Rosen 1991). In other words, he focused his attention on where the function of each component results from in the sense of Aristotle's four causal categories, that is, material cause, efficient cause, formal cause, and final cause. As a result, Rosen claimed that a material system is an organism if and only if it is closed to efficient causation.

For example, (M,R) systems (Rosen 1972) satisfy closure under efficient cause. This system model maintains its metabolic activity through inputs from environments and repair activity.

$$A \xrightarrow{f} B \xrightarrow{\phi_f} H(A,B) \xrightarrow{\Phi_f} H(B, H(A,B)) \qquad (1)$$

Here, A is a set of inputs from an environment to the system, B is a set of outputs from the system to the environment, f is a component of the system represented as a map from A to B, and ϕ_f is the repair component of f as a map from B to $H(A,B)$ ($H(X,Y)$ is the set of all maps from a set X to a set Y). In biological cells, f corresponds to the metabolism, and ϕ_f to the repair. If $\phi_f(b) = f$ ($b = f(a)$) is satisfied for the input $a \in A$, we can say that the system self–maintains itself. In addition, Φ_f can be constructed by the preceding (M, R) system in the following way: For a and b such that $b = f(a)$ and $\phi_f(b) = f$, if $\hat{b} : H(B, H(A,B)) \rightarrow H(A,B)$ ($\hat{b}(\phi)(a') = \phi(b)(a')$ ($\phi \in H(B, H(A,B)), a' \in A$)) has

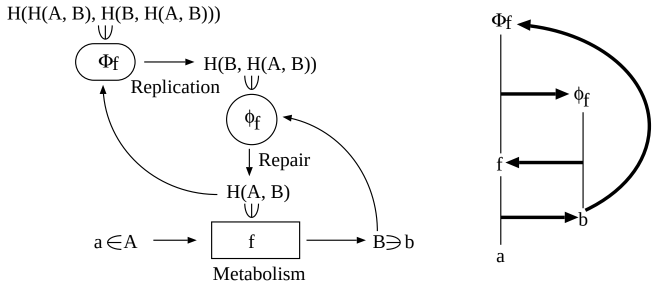

Figure 1: A (M,R) System and Its Hyperdigraph of Entailment

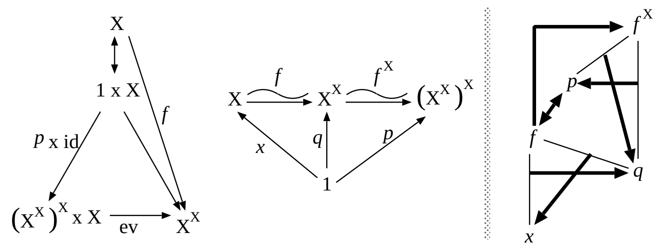

Figure 2: The Diagrams of a Completely Closed System and Its Hyperdigraph on Entailment

the inverse map $\hat{b}^{-1}$, it is easily proved that $\hat{b}^{-1}(f) = \phi_f$. Thus, we can set $\Phi_f = \hat{b}^{-1}$. The right half in figure 1 shows the aspect that the components except for a are closed under entailment, by a hyperdigraph (Higuchi et al. 1997).

It is considered that closure under entailment or production is a necessary condition for a system to be autopoietic because the components reproduce themselves in the system.

Some Systems Closed under Entailment

In order to clarify what system is closed under entailment in more general framework than the naive set theory, we use the theory of category. We assume that a category $\mathcal{C}$ has a final object 1 and product object $A \times B$ for any pair of objects A and B. The category of all sets is an example of this category. Moreover, we describe the set of morphisms from A to B as $H_{\mathcal{C}}(A, B)$ for any pair of objects A and B. A element of $H_{\mathcal{C}}(1, X)$ is called a morphic point on X. For a morphism $f \in H_{\mathcal{C}}(X, X)$ and a morphic point x on X, x is called a fixed point of f iff $f \circ x = x$ ($\circ$ means composition of morphisms)(Soto–Andrade & Varela 1984). Morphic points and fixed points are respectively abstraction of elements of a set and fixed points of maps in the category of sets.

Based on the above framework, we proposed "completely closed systems" under entailment (Nomura 2001). When components in a system are not only operands but also operators, the easiest method for representing this aspect is the assumption of existence of a isomorphism from the space of operands to the space of operators (Kampis 1991). Now, we assume an object X with powers and an isomorphism $f : X \simeq X^X$ in $\mathcal{C}$. Then, there uniquely exists a morphic point p on $(X^X)^X$ naturally corresponding to f induced in the theory of category (Takeuchi 1978). Since the morphism from X^X to $(X^X)^X$ entailed by the functor $\cdot^X$, f^X, is also isomorphic, there uniquely exists a morphic point q on X^X such that $f^X \circ q = p$. We can consider that p and q entail each other by f^X. Furthermore, there uniquely exists a morphic point x on X such that $f \circ x = q$ because f is isomorphic. Since we can consider that x and q entail each other by f, and f and p entail each other by the natural correspondence induced in the theory of category, the system consisting of x, q, p, f, and f^X is completely closed under entailment.

Generalized (M,R) Systems

We can generalize the closed part of (M,R) systems mentioned in section as follows.

For objects X and Y in $\mathcal{C}$, we assume that X has powers. When a morphism $f : X \to Y$ and a morphic point x on X are given, we assume that x satisfies the following conditions:

$$\begin{aligned} &\exists\, G_x \in H_{\mathcal{C}}(Y^X, Y) \\ &\quad s.t.,\ G_x \circ z = z' \circ x \text{ for any } z \in H_{\mathcal{C}}(1, Y^X) \\ &\text{and } G_x \text{ has its inverse morphism } F_x \in H_{\mathcal{C}}(Y, Y^X) \end{aligned} \tag{2}$$

Here, z' is the morphism from X to Y naturally corresponding to the morphic point z on Y^X. When $y = f \circ x$ and x_f is the morphic point on Y^X naturally corresponding to f, we obtain $F_x \circ y = F_x \circ f \circ x = F_x \circ G_x \circ x_f = x_f$. Thus, x_f is entailed by y and F_x. If we regard F_x as entailed by x, then f, y, F_x, and x_f are entailed by themselves and x. Although x is not entailed by x, f, y, x_f, or F_x, we can consider a larger system closed under entailment if x is one of morphic points of another closed system (for example, a completely closed system).

Infinite Regressive Systems

In this section, we consider a system like (M,R) systems including a kind of infinite regress. Now, we assume objects $X_i := {X_{i-1}}^{X_{i-2}}$, morphisms $f_i \in H_{\mathcal{C}}(X_i, X_{i+1})$, and morphic points x_i on X_i narurally corresponding to f_{i-2} and $f_i \circ x_i = x_{i+1}$ ($i = 0, \pm 1, \pm 2, \ldots$). Although any morphic point and morphism are closely entailed each other in this system, its entailment cannot be reduced to any finite subset of components and represents a kind of infinite regress. Moreover, we assume that there exist the limit X^∞ and colimit X_∞ of (X_i, f_i), and X^∞ coincides with X_∞. Furthermore, when we put $X = X^\infty = X_\infty$, and p_i and q_i are the projection from X^∞ to X_i and

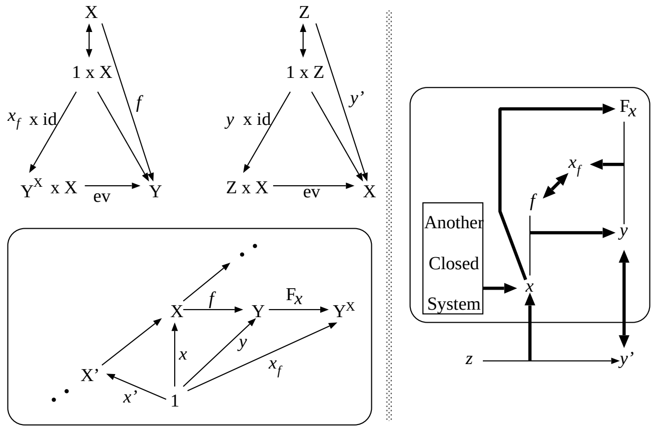

Figure 3: The Diagrams of a Generalized (M,R) System and Its Hyperdigraph on Entailment

injection from X_i to X_∞ respectively, we assume $p_i \circ q_i = \mathrm{id}_{X_i}$ $(i = 0, \pm 1, \pm 2, \ldots)$.

Then, there uniquely exists a morphic point x on X such that $p_i \circ x = x_i$ for any i since X is the limit. On the other hand, the morphic point $y = q_i \circ x_i$ (determined independent on i because of $q_i = q_{i+1} \circ f_i$) satisfies $p_i \circ y = p_i \circ q_i \circ x_i = x_i$. Thus, we obtain $x = y$ from the uniqueness of x. Moreover, when $\pi_2^{(i)}$ is the projection of $X_i \times X$ to X, the morphism g_i naturally corresponding to $\pi_2^{(i)}$ satisfies $g_i = g_{i+1} \circ f_i$. Thus, there uniquely exists a morphism $f : X \to X^X$ such that $f \circ q_i = g_i$ since X is the colimit. Figure 4 shows the diagrams of this system and its hyperdigraph on entailment.

If the above f is isomorphic, we can construct a completely closed system including f as one of its components. Then, if x is a component in this closed system, x is entailed in the system independent of $\{x_i, f_i\}$. Moreover, x_i is entailed by x and p_i, and p_i is entailed by p_{i-1} and f_{i-1}. This represents a possibility that a system consisting of infinite regress constructs a finite closed system and entailment from it by the system itself, that is, a kind of projection from the finite system to the infinite system.

Coupling of Closed Systems

Coupling is a situation that two systems are dependent each other while each system maintains its independence. In this section, we provide some forms that two systems closed under entailment in the previous section are coupling by regarding this situation as that two systems affect each other while each system maintains its closure under entailment.

First, we can consider a form of coupling of systems in the same category $\mathcal{C}$. In this case, if one of components is shared by two closed systems, it is considered that two systems affect each other while maintaining their

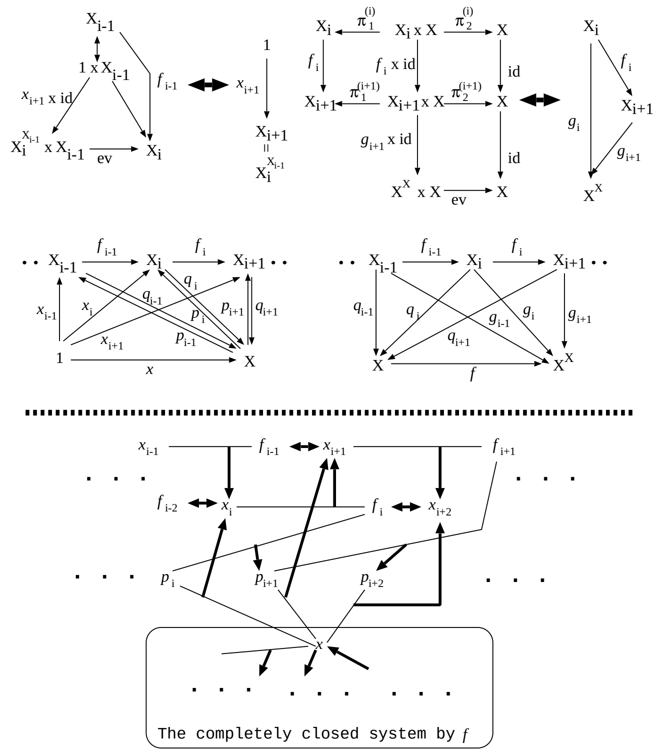

Figure 4: The Diagrams of an Infinite Regressive System and Its Hyperdigraph on Entailment

closure. Figure 5 shows this type of coupling and the hyperdigraphs on entailment. Given a completely closed system with $\{\ x,\ q,\ p,\ f,\ f^X\ \}$, and another completely closed system with $\{\ y,\ a,\ b,\ g,\ g^Y\ \}$, for example, if $X^X = Y$ and $q = y$, or $(X^X)^X = Y$ and $p = y$, these two systems affect each other while maintaining their closure.

Next, we consider a form of coupling of systems in the different categories $\mathcal{C}$ and $\mathcal{D}$. This case can represent an aspect in Luhmann's system theory, that is, the aspect that two mental systems that are independent autopoietic systems are coupled with communication system that is an autopoietic system in another level (Kneer & Nassehi 1993). Figure 6 shows this type of coupling and the hyperdigraphs on entailment. In this case, we assume a completely closed system in $\mathcal{C}$ with $\{\ x,\ q,\ p,\ f,\ f^X\ \}$, and another completely closed system in $\mathcal{D}$ with $\{\ y,\ a,\ b,\ g,\ g^Y\ \}$. Then, we assume a functor $\mathcal{F}$ from $\mathcal{C}$ to $\mathcal{D}$ such that $\mathcal{F}(1) = 1$, $\mathcal{F}(X) = Y$, $\mathcal{F}(X^X) = Y^Y$, $\mathcal{F}((X^X)^X) = (Y^Y)^Y$, $\mathcal{F}(f) = g$, $\mathcal{F}(f^X) = g^Y$, $\mathcal{F}(x) = y$, $\mathcal{F}(q) = a$, and $\mathcal{F}(p) = b$. In other words, the former system in $\mathcal{C}$ is mapped to the latter in $\mathcal{D}$ by $\mathcal{F}$. This functor exists in the level different from those where these closed systems exist, and no components in $\mathcal{C}$ and $\mathcal{D}$ entail $\mathcal{F}$. However, if there exists a subcategory of the category consisting

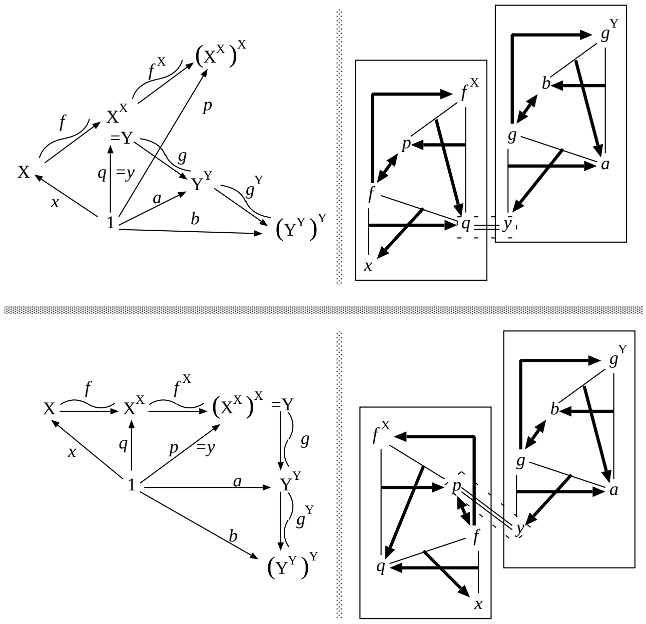

Figure 5: Coupling of Two Completely Closed Systems in the Same Category and the Hyperdigraphs on Entailment

of all the categories and functors such that it has a final object and a closed system having $\mathcal{F}$ as one of its components, $\mathcal{F}$ is entailed independent of the systems in $\mathcal{C}$ and $\mathcal{D}$.

Discussion

Although in this paper we required that operands coincide with operators ($X \simeq X^X$ or $Y \simeq Y^X$), this condition is difficult to be satisfied in the naive set theory. Although Soto–Andrade and Varela provided a category satisfying this condition (the category of partially ordered sets and continuous monotone maps with special conditions)(Soto–Andrade & Varela 1984), this category is very special. Furthermore, Rosen showed that systems closed under efficient cause cannot be described with their states because they lead to infinite regress (Rosen 1991). In addition, we needed to introduce a functor in order to connect a closed system with another system between the different categories, and needed to introduce a special category in order to entail this functor in a closed system. If these closed systems can exist only in special categories not observable in the conventional sense, autopoiesis may be hard to be a general theory of a variety of systems.

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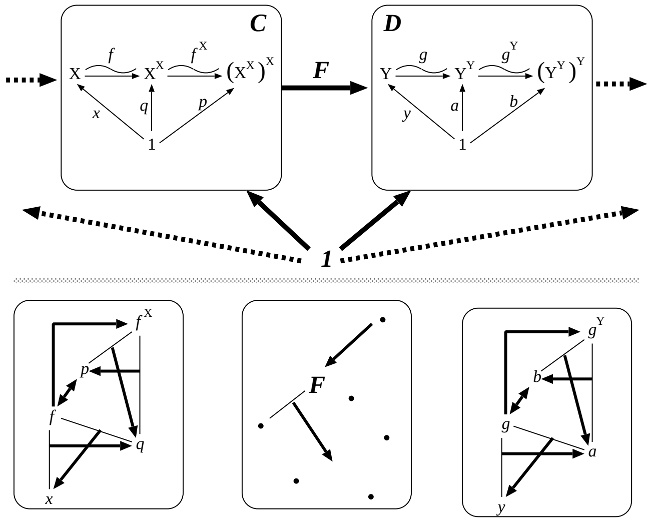

Figure 6: Coupling of Completely Closed Systems in the Different Categories and the Hyperdigraphs