Analysis and Simulation of Group Dynamics based on Interaction between Decision Making and Heider's POX Systems

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Abstract

Heider's balance theory is one of theories on micro characteristics of triad relations in social psychology. However, it has not sufficiently been discussed what relations there are between group dynamics, this micro characteristic, and other psychological processes. This paper proposes a model of interaction between the social network dynamics based on POX systems and decision making process under the human network, while comparing the proposed model with the previous group dynamics model based on only POX systems.

1 Introduction

As one of theories on micro characteristics of individuals in social psychology, balance theory proposed by F. Heider [1] states a psychological stability of an individual included in a triad relation. In this theory, a person (P), another person (O), an object or the third person (X), and relations from P to O, from O to X, and from P to X construct a system (called POX system). These relations have either $+$ or $-$ value corresponding to the fact that the person likes or dislikes the object respectively. Heider's theory argues that a POX system is balanced if and only if the product of the signs on these three relations is $+$, and if the system is not balanced P changes one of the relations to O and X so that the POX system becomes balanced. As shown in Figure 1, if the system is not balanced, then P inverts either the sign of $P \to O$ or that of $P \to O$ to balance the POX system.

Although the original balance theory is limited to triad relations, its extension to groups consisting of more than three persons have been proposed [2, 3, 4]. These studies of balance in social networks focus on network structures of balanced situations based on graph theory.

However, it has not sufficiently discussed what structures actually appear in large groups as a macro structure of group dynamics based on micro behaviors of the original POX systems in individual persons. As an approach to this problem in the field of artificial societies, Wang and Thonegate [5] proposed a simulation model of group dynamics based on POX systems, consisting of full connected graphs. However, this study focuses on non–digraphs, that is, cases where all the dyad relations are symmetric. We proposed a formalization of group dynamics based on POX systems as a finite Markov chain with a state space consisting of signs on all the edges in digraphs, characterized the concept of balance as absorbing states of this Markov chain, and executed computer simulations of the group dynamics based on the Markov chain [6].

However, the model by Wang and Thonegate and our model focus on group dynamics only based on POX systems, and they lack interaction between POX systems and other psychological processes. To construct more realistic models of group dynamics, this paper proposes a model of interaction between the social network dynamics based on POX systems and decision making process under the human network.

2 Group Dynamics as a Finite Markov Chain

In the same way as the previous work [6], we assume that there are N persons and relations between them are represented as a fully connected digraph in which these relations have $+$ or $-$ value, where $+$ and $-$ mean that the person likes and dislikes the other person, respectively. In addition, it is assumed that these individuals make decisions for a subject by agreement ($+$) or disagreement (-).

This social network can be represented as a signed digraph $G = (P, A)$, $P = \{p_1, p_2, \ldots, p_N, s\}$, $A = \{(a_{12}, s_{12}), (a_{13}, s_{13}), \ldots, (a_{21}, s_{21}), (a_{23}, s_{23}), \ldots, (a_1, s_1), \ldots, (a_N, s_N)\}$. P is the set of $(N+1)$ vertices

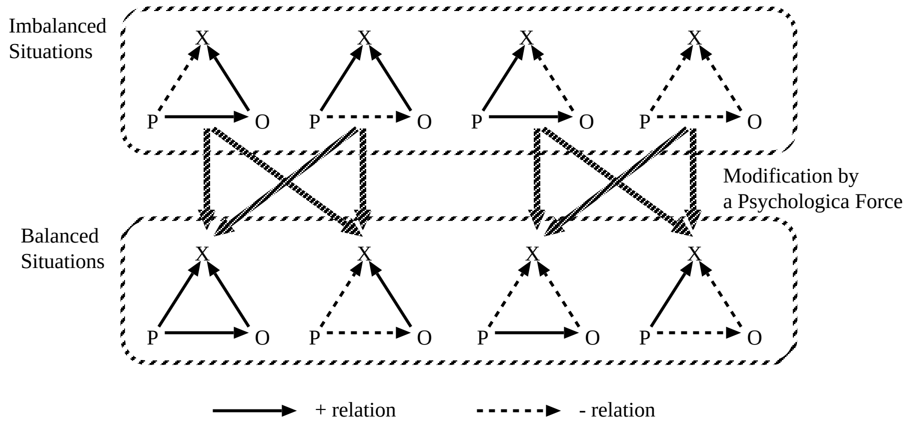

Figure 1: Balanced and Imbalanced Situations of POX Systems in Heider's Balance Theory.

in the digraph, corresponding to the N persons and the subject s, and A is the set of pairs of edges a_{ij} from the i–th person to the j–th person and signs on them s_{ij}, and pairs of edges a_i from the i–th person to the subject s and signs on them s_i.

In this model, each individual synchronously does the following actions:

1. It selects one of the POX systems including itself (these POX systems include the triads consisting of the edge to the subject). If the selected POX system is imbalanced, it balances the POX system by randomly selecting any of the edges toward the others and reversing the sign on the selected edge (if the selected POX system includes the subject, the sign of the edge toward the other is changed and that toward the subject is not changed).

2. Among the POX systems including the subject, if the number of the imbalanced ones is larger than that of the balanced ones, the sign of the edge toward the subject is reversed.

The above procedure 1 represents balance of the POX systems in each individual. The above procedure 2 represents decision making of the individual based on subordination to a majority and balance of POX systems. The number of the balanced POX systems including the subject is the sum of the total of the number of the others having the same opinion as the individuals and liked by the individual, and the number of the others having the opposite opinions to the individual and disliked by the individual. Moreover, the number of the imbalanced POX systems including the subject is the sum of the total of the number of the others having the opposite opinion to the individuals and liked by the individual, and the number of the others having the same opinions to the individual and disliked by the individual. Thus, reversion of the sign toward the subjects means reversion of these numbers, and the procedure represents decision making for the subject based on minimization of the imbalanced POX systems including the subject.

The change of signs from individuals to others in the above procedure 1 is stochastic and dependent only on the current signs, although the change of signs of individuals toward the subject is deterministic. Thus, group dynamics based on these procedures equals to a finite Markov chain with the state space $S = \{(s_{12}, s_{13}, \ldots, s_{21}, s_{23}, \ldots, s_1, \ldots, s_N)\}$, in which the total number of states is 2^{N^2}. Figure 2 shows an example of this finite Markov chain with 3 persons.

Relations between Absorbing States and Balanced Situations

Absorbing states in the above finite Markov chain as group dynamics are situations where all the POX systems are balanced. It is shown that these situations coincide with the following situations of the group: (1) the group is partitioned into two subgroups (including the case that one of the subgroups is empty), (2) all the relations between individuals in the same subgroup are positive, (3) all the relations between individuals in the different subgroups are negative, (4) the individuals in the same subgroup have the same attitude toward the subject, and those in the different subgroup have the opposite attitude.

It is trivial that in the situations satisfying the above four conditions all the POX systems are balanced. The converse is proved as follows.

First, all the POX systems consisting of N–persons are balanced. In the previous work [6], it was proved that this condition is a necessary and sufficient con-

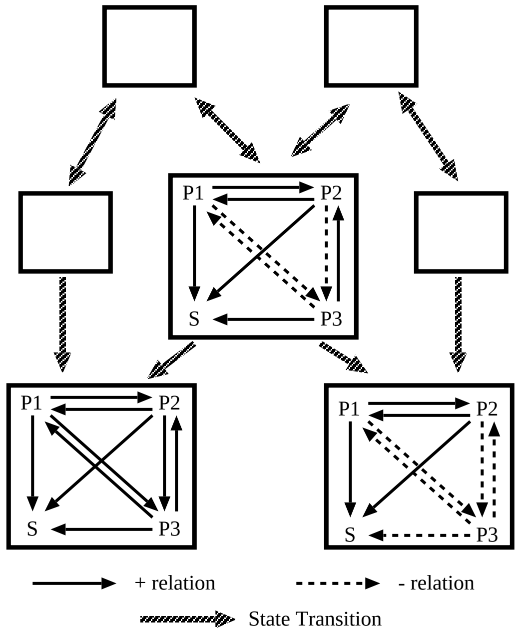

Figure 2: An Example of the Finite Markov Chain of the Group Dynamics (3 Persons)

dition for the above conditions (1)–(3). In fact, when all the POX systems consisting of N–persons are balanced, it is proved that the relation between persons $p_i \sim p_j \stackrel{\text{def}}{=} \{p_i = p_j$ or the edge $p_i \rightarrow p_j$ is $+\}$ is an equivalence relation. For this equivalence relation $\sim$, persons in the same equivalence class are connected each other by edges with + and vertices in the different classes are connected each other by edges with $-$. Since balance of all the POX systems does not permit the existence of more than three classes, this situation equals to the above conditions 1–3.

Then, if persons in the same subgroup have the different attitudes toward the subject, the POX systems consisting of these persons and the subject are imbalanced since the relations between these persons are positive. Moreover, if persons in the different subgroups have the same attitude toward the subject, the POX systems consisting of these persons and the subject are imbalanced since the relations between these persons are negative. Thus, the above condition (4) must be satisfied.

3 Simulation

As shown in the previous section, the group dynamics based on individual POX systems and decision making is represented as a finite Markov chain having absorbing states corresponding to situations where the group is polarized into two subgroups. However, this analysis does not clarify whether this finite Markov chain has cyclic states. In other words, there is a possibility of the existence of cyclic states where modification of some POX systems and that of other POX systems are repeated one another.

In order to investigate the above problem, we executed some computer simulations. The simulations were executed for configurations of $N = 4$–8. For each configuration, 300 trials with different random seeds were executed. For each trial, the probability of selection of each POX system in the procedure 1 mentioned in section 2 were fixed as follows: $\frac{1}{2(N-1)(N-2)}$ for the POX system not including the subject, and $\frac{1}{2(N-1)}$ for the POX systems including the subject (note that the total number of POX systems including the subject and that not including it for a person are $N-1$ and $(N-1)(N-2)$ respectively).

Table 1 shows types of grouping in absorbing states and the numbers of the corresponding states, the numbers of trials that converged to the corresponding states, and mean number of iteration for convergence to each grouping. The state converged to one of absorbing states shown in the previous section in all the trials for all the configurations, and any cyclic state was not observed. Moreover, no trend existed that there is a specific absorbing state to which convergence is faster than the other states.

Comparison with the Previous Model

In the previous work [6], we proposed the group dynamics model based on only POX systems, without the decision making process. This previous model showed the analysis and simulation results similar to the model proposed in section 2. However, there are some phenomena different between the previous and proposed model.

The absorbing states of the previous model are also situations where the group is polarized into two subgroups, including the case that one of them is empty, and the simulation results showed that the state converged to one of absorbing states in all the trials. However, the speed of convergence to the absorbing states in the proposed model was much faster than that in the previous model. Figure 3 shows means of iteration numbers spent until convergence for each N in the proposed and previous models. For increase of the number of persons, the convergence speed of the proposed model increased linearly, although that of the previous model increased exponentially.

Furthermore, there was a bias on which absorbing state the group converged to, although this bias was

Table 1: Results of Simulations

N	4			5			6			
Types of Grouping (#. Corresponding States)	4 : 0 (1)	3 : 1 (4)	2 : 2 (3)	5 : 0 (1)	4 : 1 (5)	3 : 2 (10)	6 : 0 (1)	5 : 1 (6)	4 : 2 (15)	3 : 3 (20)
#. Convergence	82	149	69	57	116	127	43	92	110	55
Mean #. Iteration	14.2	12.8	13.7	37.6	29.4	24.2	41.3	54.2	47.5	46.1

N	7				8					
Types of Grouping (#. Corresponding States)	0 : 7 (1)	1 : 6 (7)	2 : 5 (21)	3 : 4 (35)	0 : 8 (1)	1 : 7 (8)	2 : 6 (28)	3 : 5 (56)	4 : 4 (35)	
#. Convergence	18	67	112	103	17	61	114	71	37	
Mean #. Iteration	81.7	65.5	63.1	63.1	111.9	123.7	102.8	98.0	97.1	

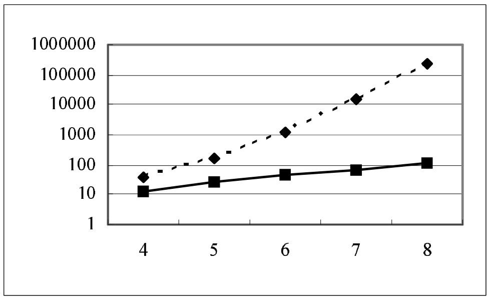

Figure 3: Means of Iteration Numbers Spent Until Convergence (Solid Line: The Proposed Model, Dashed Line: The Previous Model without Decision Making Process)

not observed in the previous model. For example in $N = 4$, the situation where all the persons like each other appeared 82 times among 300 trials, although the polarized group situations appeared about 20–40 times in average. In the same simulations of the previous model, the situation where all the persons like each other appeared 29, 16, 7, 5, 2 times among 300 trials for $N = 4, 5, 6, 7, 8$ respectively. For each N, a χ^2–test revealed that there was a statistically significant difference with 0.001 or 0.01 level on these numbers of convergence between the proposed model and previous model.

The above facts suggest that the decision making process has an effect of acceleration and bias in convergence of the group dynamics based on micro behaviors of POX systems.

4 Summary

This paper proposed a model of interaction between the social network dynamics based on POX systems and decision making process under the human network, while comparing the proposed model with the previous group dynamics model based on only POX systems.

As future problems, we should investigate a cause of observed acceleration and bias in convergence of the proposed model in comparison with the previous model while exploring the corresponding psychological phenomena. In addition, we need to extend the full–connected graph structure of the model to general graphs including 2–D cellular structures.

Acknowledgments

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References

[1] Heider, F. (1958), *The Psychology of Interpersonal Relations*. John Wiley & Sons. (Japanese Edition: M. Ohashi, Seishin, 1978).

[2] Cartwright, D., Harary, F. (1956), Structural balance: A generalization of Heider's theory. *Psychological Review*, 63: 277–293.

[3] Harary, F., Norman, R. Z., Cartwright, D. (1965), *Structural Models: An Introduction to the Theory of Directed Graphs*. John Wiley & Sons.

[4] Flament, C. (1963), *Applications of Graph Theory to Group Structure*, Printice–Hall. (Japanese edition: K. Yamamoto, Kinokuniya, 1974).

[5] Wang, Z., Thorngate, W. (2003), Sentiment and social mitosis: Implications of Heider's balance theory. *Journal of Artificial Societies and Social Simulations*, 6(3). (electric publication: `http://jasss.soc.surrey.ac.uk/6/3/2.html`).

[6] Nomura, T. (2004), Analysis and simulation of group dynamics based on Heider's balance theory and a finite Markov chain. In *Proc. 2nd International Conference of the European Social Simulation Association*. (CD–ROM Proceedings).